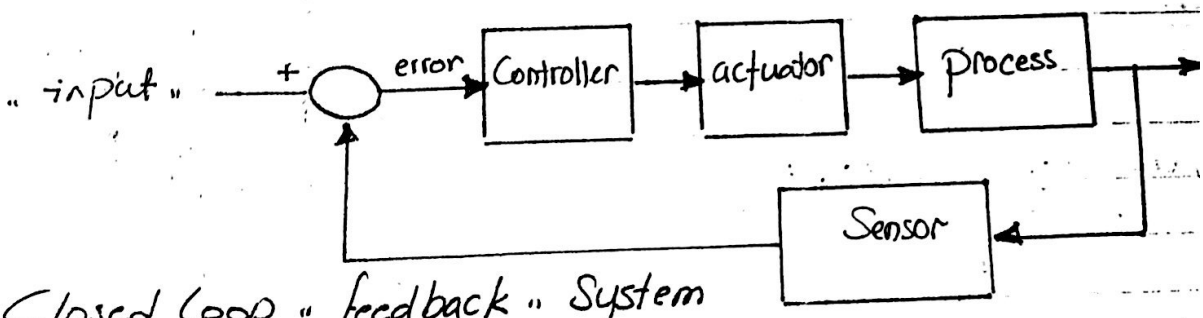
EE362

Lecture #

* Definition :— purpose of obtaining desired Output with desired performance
Consists of subsystems and process(plant) assembled for the
Control System is One where the Output of the System is
Controlled to be at some specific values (regulation), or
Changes in some prescriped way (tracking) as determined
by the System.

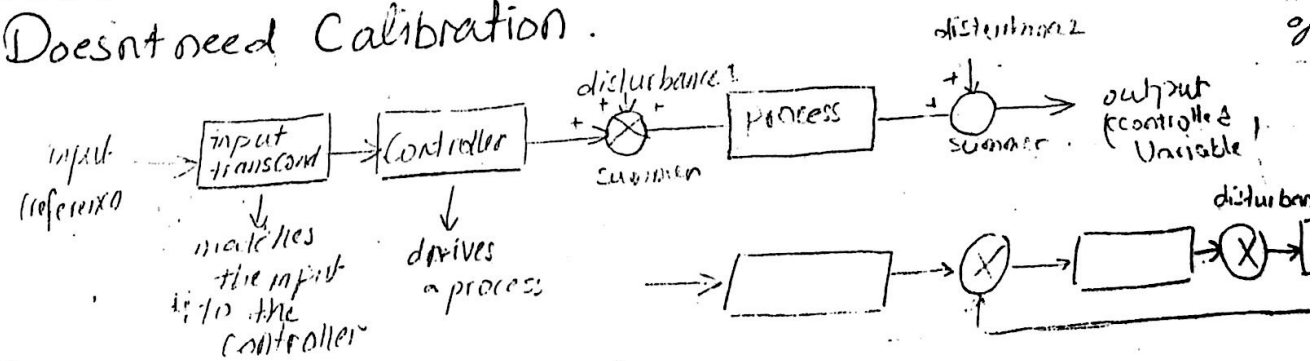
* Components of the Control system :—
1. process (plant):
2. Actuator.
3. Sensor (measurement element)
4. Controller.



Closed Loop "feedback" System

* Advantages :—
- Accuracy
- Disturbance Rejection
- Stablize non-stable system
- Doesnt need Calibration.

* dis advantages :—
- more complicated
- Output Should be mea
- Can go Unstable...

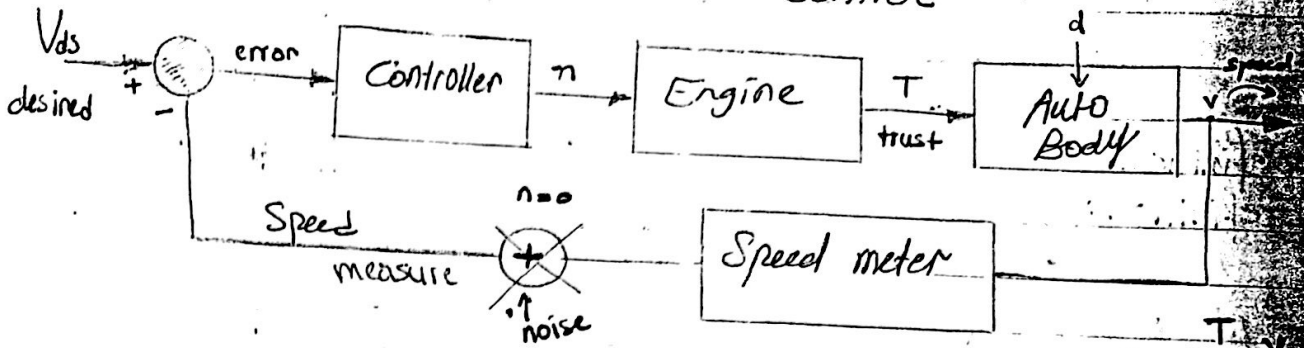


1.53

Scanned by CamScanner

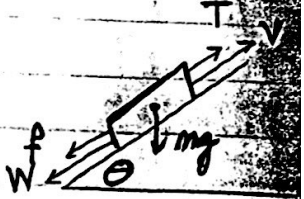
* Example (1) :-

Cruise Control



Mathematical model :-

$$m\dot{V} = T - F - W$$



assume: $F = bV$ → friction.

$W = mg \sin\theta$ → force due to road grade. (تدرج)

$T = au$ → throttle angle.

* at Steady State : $\dot{V} = 0$ $(d/dt = \dot{V})$

but :- $m\dot{V} = au - bV - w$ → (1)

m, a, b, w → are uncertain

Open loop Control :- (تحكم) العلاقات غير معتمدة على قيمة

to Calculate Control assuming : $w = 0$, $m = \hat{m}$, $b = \hat{b}$

nominal Values : (قيم اسمية)

$\hat{m}\dot{V} = au - \hat{b}V_{ds} = 0$ at Steady State

$u = \frac{\hat{b}}{a} V_{ds}$ → سرعة معينة

Analysis :- apply the Control (u) to the System and assume that $w \neq 0$ (constant) , $b \neq \hat{b}$, $m \neq \hat{m}$

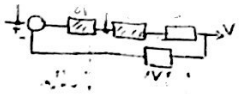
in (1) ⇒ $m\dot{V} = a\left(\frac{\hat{b}}{a} V_{ds}\right) - bV - w$

at Steady State $\dot{V} = 0$

$$V = \frac{\hat{b}}{b} V_{ds} - \frac{1}{b} W$$

Scanned by CamScanner

* Closed Loop :-



We can design U as : $U = (K)(V_{ds} - V) + \left(\frac{\hat{b}}{\hat{a}}\right) V_{ds}$ ⇒ High Gain feed back

feed forward

$m \neq \hat{m}$, $b \neq \hat{b}$, $w \neq 0$

in (1) → $mV = ak(V_{ds} - V) - bV - W + \hat{b} V_{ds}$

at Steady state :-

$$(ak + \hat{b}) V_{ds} - (b + ak) V - W = 0$$

$$V = \frac{ak + \hat{b}}{ak + b} V_{ds} - \frac{1}{b + ak} W$$

$\frac{ak + \hat{b}}{ak + b} \to 1$, $\frac{1}{b + ak} \to 0$

ask grows → V approach $V_{desired}$.

5th March 2016.

* Laplace transform :-

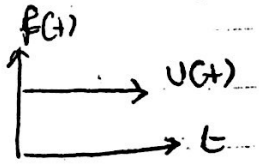
Mathematical method which can be used to convert differential equations into algebric equations in S domain.

it's easier to get the Solution in (S).

Dif :- $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt = f(S)$.

$s = \sigma + j\omega$.

Example (1) :- find the laplacian transformation (L.T) Step Unite function.



$$\int_0^\infty 1 \cdot e^{-st}\, dt = \frac{e^{-st}}{-s} \Big|_0^\infty = 0 - \left(\frac{1}{-s}\right) = \frac{1}{S}$$

* $\mathcal{L}(KU(t)) = \frac{K}{S}$

" inverse "

$$\mathcal{L}^{-1}(F(t)) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, dS$$

Scanned by CamScanner

* Example(2):- find (L.T) for $f(t) = e^{at}$ $a \geq 0$

$$\int_0^\infty e^{at} e^{-st}\, dt = \int_0^\infty e^{-(s-a)t}\, dt = \frac{e^{-(s-a)t}}{-(s-a)}\Big|_0^\infty = \frac{1}{s-a}$$

Rules for laplace Transforms:—

Sum of transformation

$$f_1(t) \pm f_2(t) \leftrightarrow f_1(s) \pm f_2(s)$$

Multiplication by Constant (A)

$$A f(t) \leftrightarrow A f(s)$$

Shifting in time, (T) delay:

$$f(t-T) \leftrightarrow f(s) e^{-Ts}$$

first divitive

$$\frac{d}{dt} f(t) \leftrightarrow s f(s) - f(0)$$

Second divitive

$$\frac{d^2}{dt^2} f(t) \leftrightarrow s^2 f(s) - s f(0) - \frac{d}{dt} f(0)$$

n^{th} divitive

$$\frac{d^n}{dt^n} f(t) \leftrightarrow s^n f(s) \dots \frac{d^{n-1}}{dt^{n-1}} f(0)$$

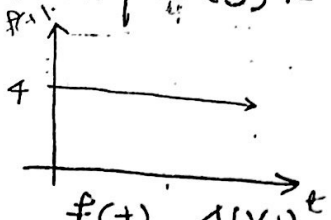
Integration

$$\mathcal{L}\left(\int f(t)\, dt\right) = \frac{f(s)}{s}$$

$$\mathcal{L}\left(\iint \dots \int f(t)\, dt^n\right) \leftrightarrow \frac{f(s)}{s^n}$$

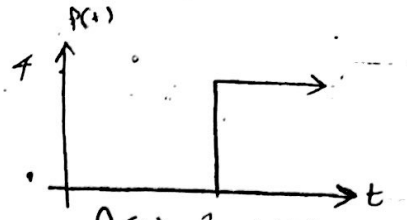
Scanned by CamScanner

Example (3):- Find F(S):-



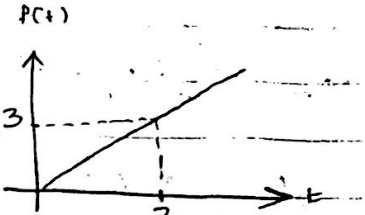
$f(t) = 4U(t)$

$f(S) = 4/S$



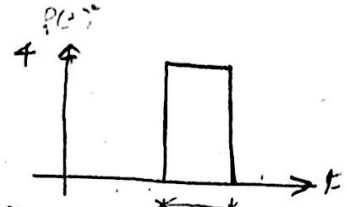
$f(t) = 4U(t-2)$

$f(S) = \frac{4}{S} e^{-2S}$



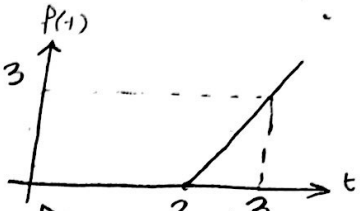
$f(t) = \frac{3}{2} r(t)$

$f(S) = 3/S^2$



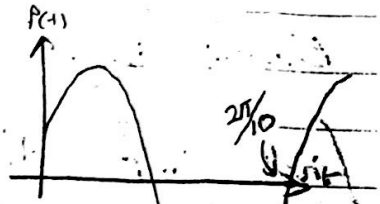
$f(t) = 4\Pi(t-2)$

$F(S) = 4e^{-2S}$



$f(t) = r^2(t)^3$

$f(S) = \frac{3}{S^2} e^{-2S}$



$F(S) = \frac{\omega}{S^2+\omega^2}$ → ω = 10

$F(S) = \frac{2\times10}{S^2+100}$

Example (4):- Find f(S):-

$t^2 \rightarrow F(S) = 2!/S^3$.

$t^2 e^{-at} \rightarrow F(S) = 2!/(S+a)^3$.

$t^2(1+e^{-at}) \rightarrow F(S) = \frac{2}{S^3} + \frac{2}{(S+a)^3}$. #

Example (5):- Solve Using Laplace transform. XC

$\frac{3dx}{dt} + 2x = 4$

$3S\,X(S) + 2\,X(S) = \frac{4}{S} \rightarrow X(S) = \frac{4}{S(3S+2)}$

from table: $A/S(S+A) \leftrightarrow 1 - e^{-at}$.

$X(S) = \frac{4/3}{S(S+2/3)} \cdot = 2 \cdot \frac{2/3}{S(S+2/3)}$

$x(t) = 2(1 - e^{2/3 t})$.

U(t): shows that the response is zero until $t=0$

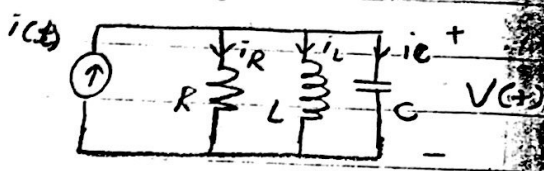
Unless otherwise was specified = all inputs wont start Until $t=0$.

Scanned by CamScanner

* Transfer function :- $G(s) = \frac{C(s)}{R(s)} = \frac{\text{pol}(B)}{\text{Pol}(a)}$

It's the ratio of the Laplace transform of the output to the laplace transform of the input (assuming zero initial condition).

Example (1) :-



find T.f $\frac{V(s)}{I(s)}$

Method (1) :- Using differential equations.

$$i(t) = i_R + i_L + i_C$$

$$= \frac{V(t)}{R} + \frac{1}{L}\int_0^t V(t)\,dt + C\frac{dV(t)}{dt} + i_L(0^-)$$

(0 → initial condition)

$$\frac{di(t)}{dt} = \frac{1}{R}\frac{dV(t)}{dt} + \frac{1}{L}V(t) + C\frac{d^2V(t)}{dt^2}$$

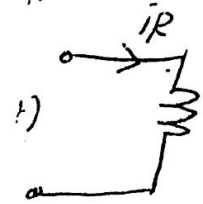
taking Laplacian transform for both Sides.

$$S\,I(s) = \frac{S}{R}V(s) + \frac{1}{L}V(s) + CS^2V(s)$$

$$\frac{V(s)}{I(s)} = \frac{S}{CS^2 + \frac{1}{R}S + \frac{1}{L}}$$

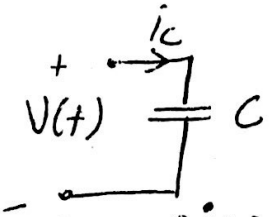
Method (2) :-

* We know that :-



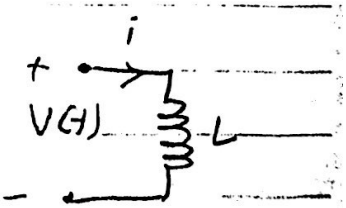
$V = IR$

$V(s) = I(s)R$



$V(t) = \frac{1}{C}\int i(t)\,dt$

$V(s) = \frac{1}{C}I(s)$



$V(t) = L\,di/dt$

$V(s) = LSI(s)$

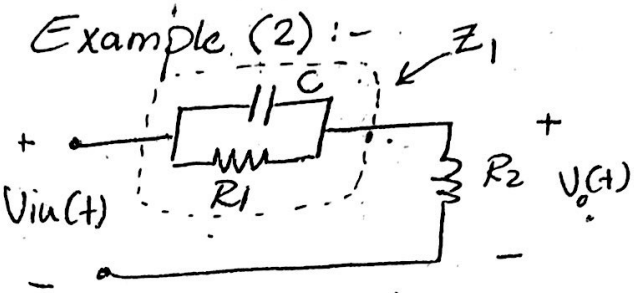
Scanned by CamScanner

Impedance :- $Z(s) = \frac{V(s)}{I(s)}$

Resistance → R
Capacitor → $\frac{1}{Cs}$
inductor → Ls

$$I(s) = I_R + I_L + I_C$$
$$I(s) = \frac{V(s)}{R} + \frac{V(s)}{Ls} + Cs\,V(s)$$
$$\frac{V(s)}{I(s)} = \frac{1}{1/R + 1/Ls + Cs}$$

Example (2) :-

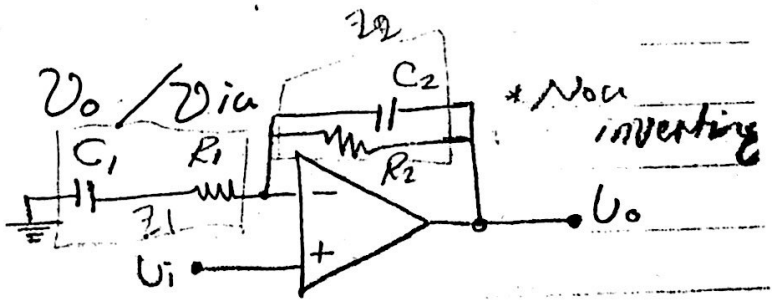


find T.f

$$\frac{V_o(s)}{V_{in}(s)} = \frac{R_2}{Z_1 + R_2}$$

$$Z_1 = \frac{R_1/Cs}{R_1 + \frac{1}{Cs}} = \frac{R_1}{R_1Cs+1} \rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{R_2(SCR_1+}{R_1 + R_2(CSR_1+}$$

Example (3) :- find V_o/V_{in}



* Non inverting

$$\frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1} = 1 + \frac{Z_2}{Z_1}$$

$$\frac{V_o}{V_i} = 1 + \frac{R_f}{R_1} = 1 + \frac{Z}{Z_1}$$

$$Z_1 = R_1 + \frac{1}{C_1 S} = \frac{R_1C_1S+1}{C_1S}$$

$$Z_2 = \frac{R_2 \times 1/C_2S}{R_2 + 1/C_2S} = \frac{R_2/C_2S}{R_2C_2S/C_2S} = \frac{R_2}{R_2C_2S+1}$$

inverting :-

$$\frac{V_o}{V_i} = \frac{-R_f}{R_1}$$

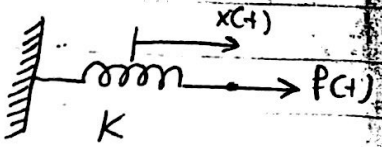
$$\frac{V_o}{V_i}(S) = 1 + \frac{1}{\left(\frac{R_1C_1S+1}{C_1S}\right)C_2S} = 1 + \frac{1}{(R_1C_1S+1)C_2}.$$

Scanned by CamScanner

* Transfer function (mechanical systems):-

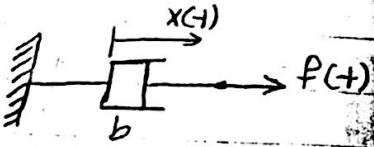
* Translational mechanic system:-

1). Spring :-



$$f(t) = K\,x(t)$$
$$F(s) = K\,x(s)$$

2). Viscous damper :-



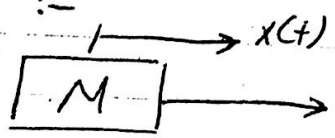
$$f(t) = b\,\frac{dx(t)}{dt} \qquad F(s) = bsX(s)$$

Also friction

b ; damping constant

.. ≡ friction factor.

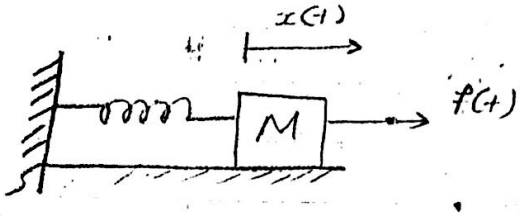
3). Mass :-



$$f(t) = M\frac{dx^2(t)}{dt^2}$$

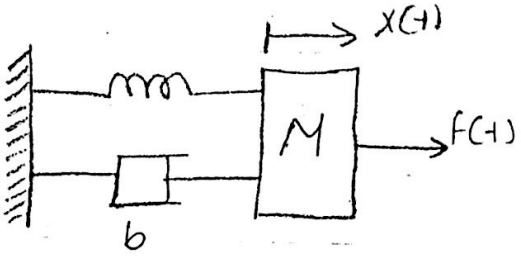
$$F(s) = Ms^2X(s)$$

Example (1):- Find T.f (X/F)

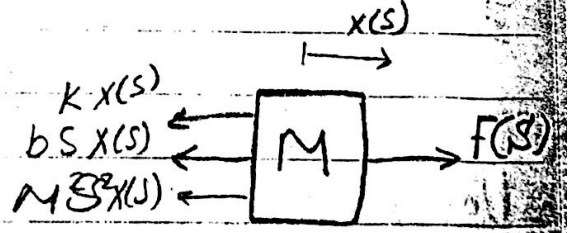


$$F = kX + bsX + Ms^2X$$

$$\frac{X}{F} = \frac{1}{Ms^2 + bs + k}$$



* Free body Diagram:-



Scanned by CamScanner

HW(1)

Find $x(t)$ for the following :—

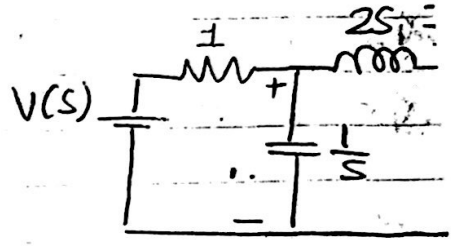
① $\frac{4S-2}{S^2-2S-8}$ ② $\frac{3S^2+10S-2}{S^2+3S+2}$ ③ $\frac{4(S+45)}{S(S^2+6S+45)}$

④ $\frac{S^2+4S+1}{(S+2)(S+1)^2}$

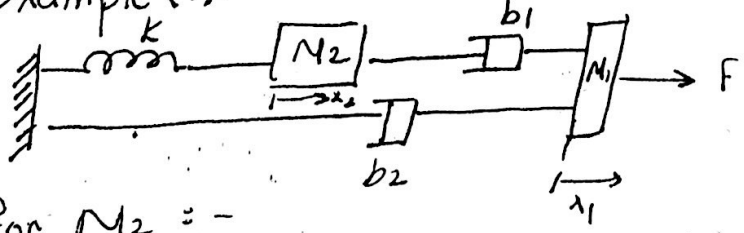
Use Matlab to find the partial fraction.

$$I(S) = \frac{V_1}{2S+1}$$

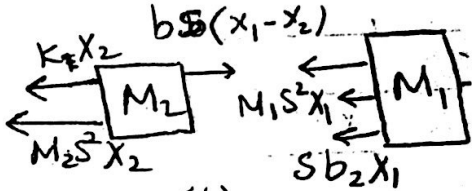
$$\frac{V_1}{V(S)} = (2S+1)/(2S^2+S+1)$$



Example (1):-



find x_1/F.



*for M_2 :-

$$KX_2 + M_2S^2X_2 - b_1S(X_1 - X_2) = 0$$

$$-bSX_1 + X_2(M_2S^2 + b_1S + K) = 0 \quad \rightarrow (1)$$

*for M_1 :-

$$M_1S^2x_1 + b_1S(x_1 - x_2) + Sb_1x_1 = F \quad (1)$$

$$(M_1S^2 + (b_1+b_2)S)x_1 - b_1Sx_2 = F \quad \rightarrow (2)$$

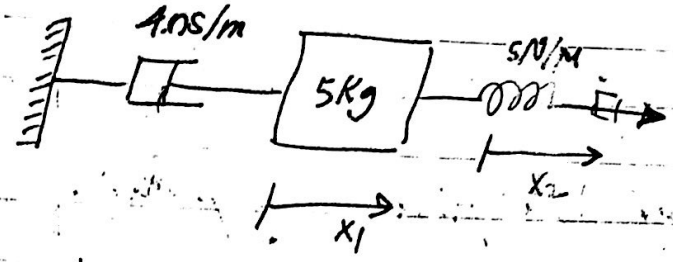
$$x_1 = \frac{\begin{vmatrix} F & -b_1S \\ 0 & M_2S^2+b_1S+K \end{vmatrix}}{\begin{vmatrix} M_1S^2(b_1+b_2)S & -b_1S \\ -b_1S & M_2S^2+b_1S+K \end{vmatrix}} = \frac{(M_2S^2+b_1S+K)F}{\Delta S}$$

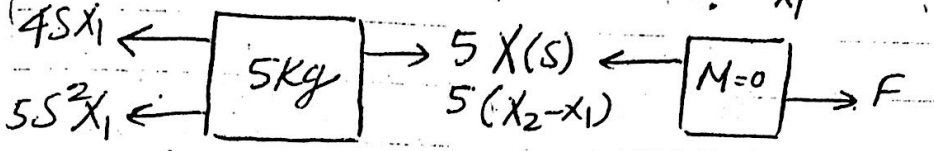
Scanned by CamScanner

$$\frac{X_1}{F} = \frac{M_2 S^2 + b_1 S + K}{\Delta S}$$

$$\Delta S = (M_1 S^2 + (b_1 + b_2) S)(M_1 S^2 + b_1 S + K) + (b_1 S)^2$$

Find $\frac{X_1}{F}$





$$4SX_1 + 5S^2X_1 - 5(X_2 - X_1) = 0$$

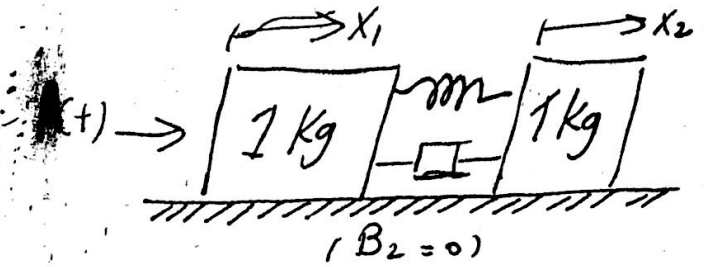
$$(5S^2 + 4S + 5)X_1 - 5X_2 = 0 \quad \rightarrow (1)$$

$$5(X_2 - X_1) = F(S) \quad \rightarrow (2)$$

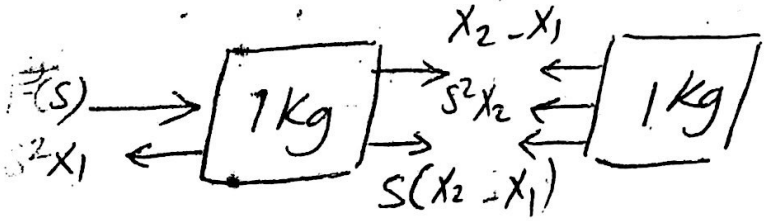
adding (1) to (2)

$$(5S^2 + 4S)X_1 = F(S)$$

$$\frac{X_1}{F} = \frac{1}{5S^2 + 4S} \quad \#$$



Find $\frac{X_2}{F}$



Scanned by CamScanner

$$S^2 x_1 - (x_2 - x_1) - S(x_2 - x_1) = F$$

$$(S^2 + S + 1) X_1 - (S+1) X_2 = F \quad \rightarrow (1)$$

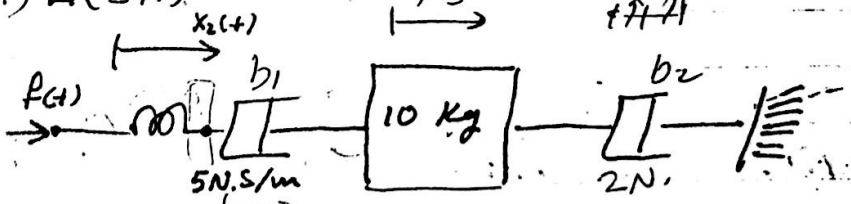
$$(X_2 - X_1) + S(X_2 - X_1) + S^2 X_2 = 0$$

$$-(S+1) X_1 + (S^2 + S + 1) X_2 = 0 \quad \rightarrow (2)$$

$$X_2 = \frac{\begin{vmatrix} S^2+S+1 & F \\ -(S+1) & 0 \end{vmatrix}}{\begin{vmatrix} S^2+S+1 & -(S+1) \\ -(S+1) & (S^2+S+1) \end{vmatrix}} = \frac{F(S+1)}{(S^2+S+1)^2 - (S+1)^2}$$

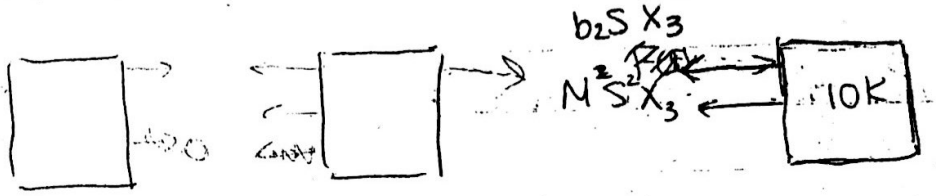
$$\frac{X_2}{F} = \frac{(S+1)}{(S^2+S+1)^2 - (S+1)^2}$$

Exercise



- Determine where to derate x_1 x_1, x_3
- find x_1 / f

result : $\left[\frac{10}{S(S^2 + 50S + 2)} \right]$

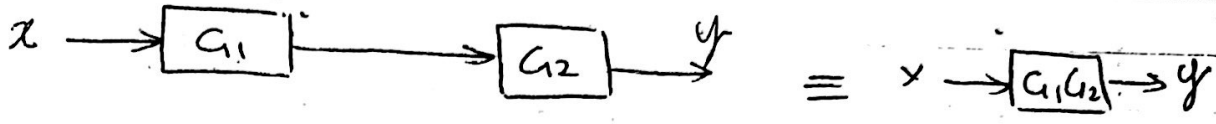


Scanned by CamScanner

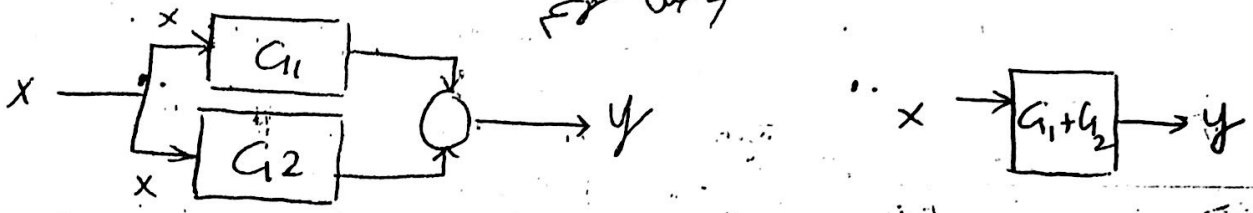
15th march 2016

* Block Diagram Reduction :-

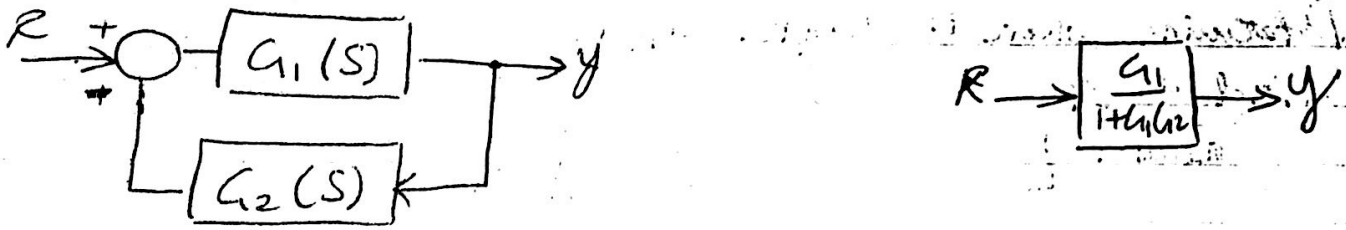
* Series Connection :



* Parallel Connection :-

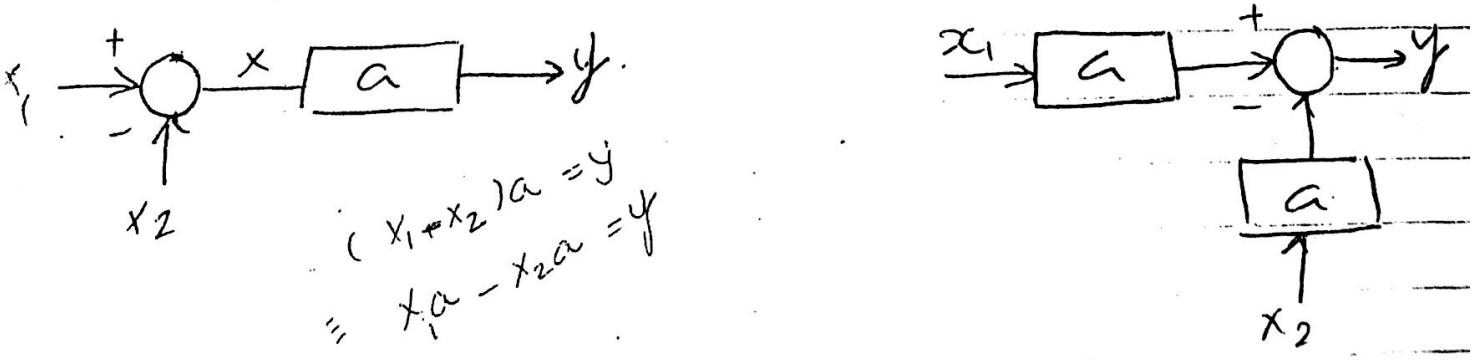


* Negative feedback :-



* Posetive feed back :-

* Moving Block :-

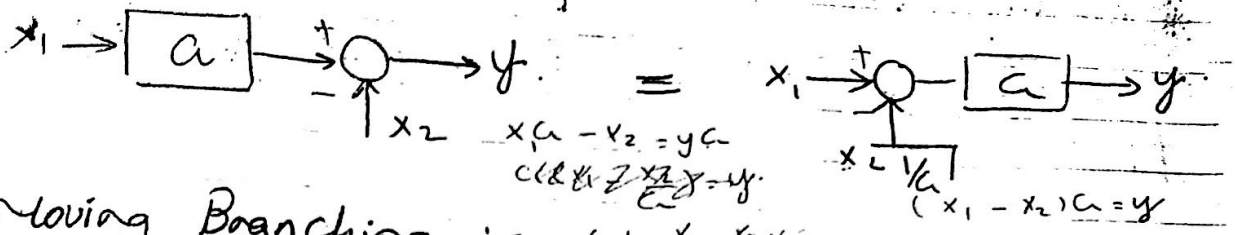


$(x_1 - x_2)a = y$

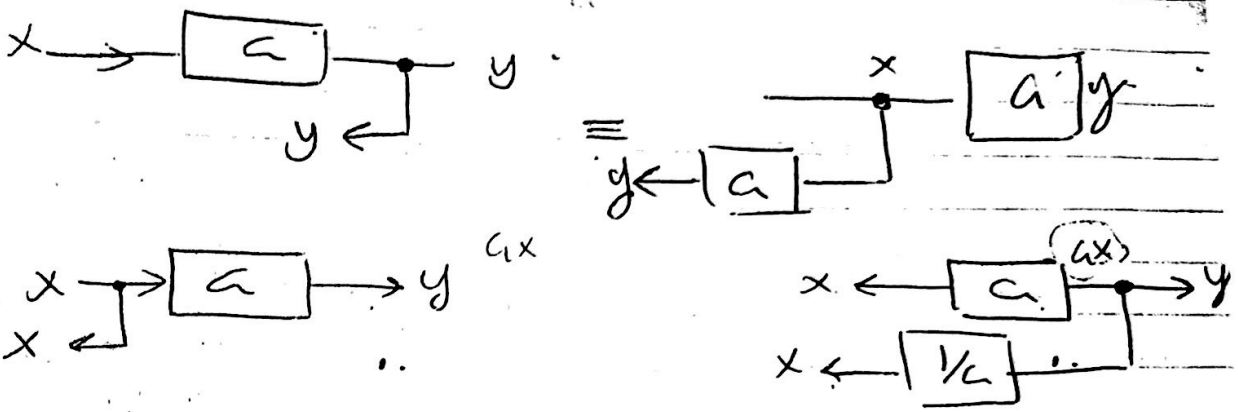
$x_1 a - x_2 a = y$

Scanned by CamScanner

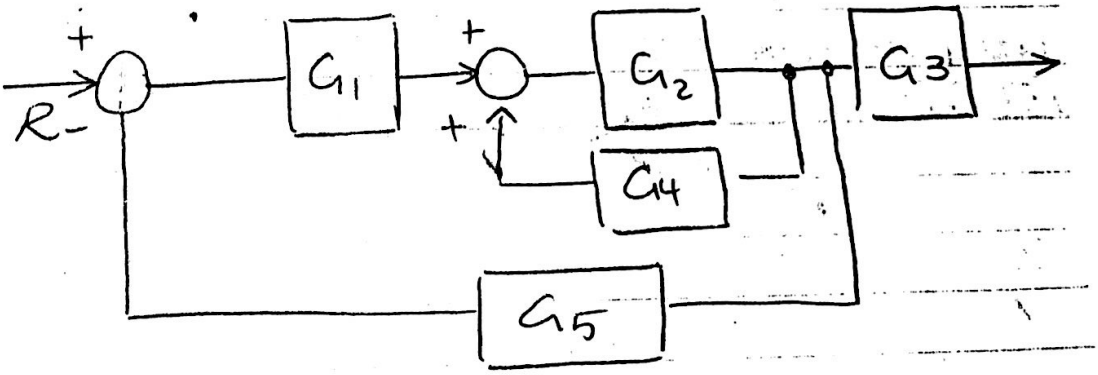
* Moving of Summing Point :-

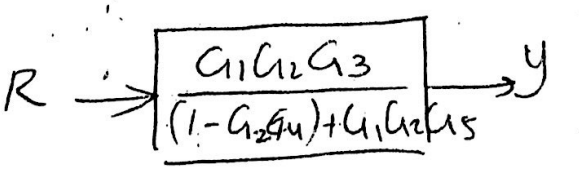


Moving Branching :-

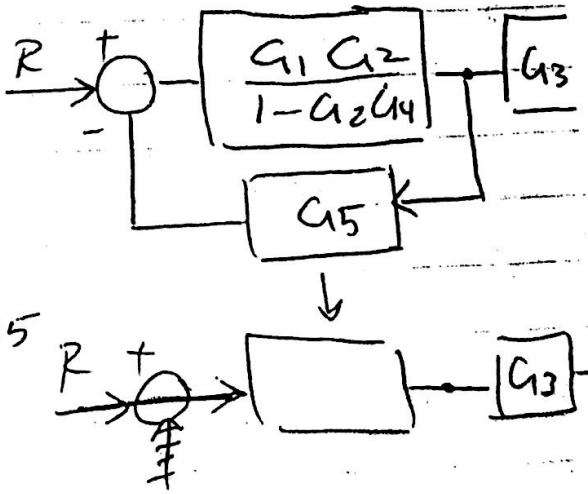


* Example (1) :- Find $G = y/R$

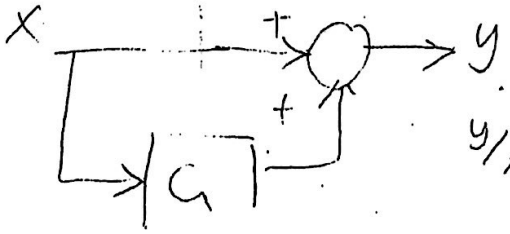




⇐



$$\frac{y}{R} = \frac{G_1 G_2 G_3}{(1 - G_2 G_4) + G_1 G_2 G_5}$$

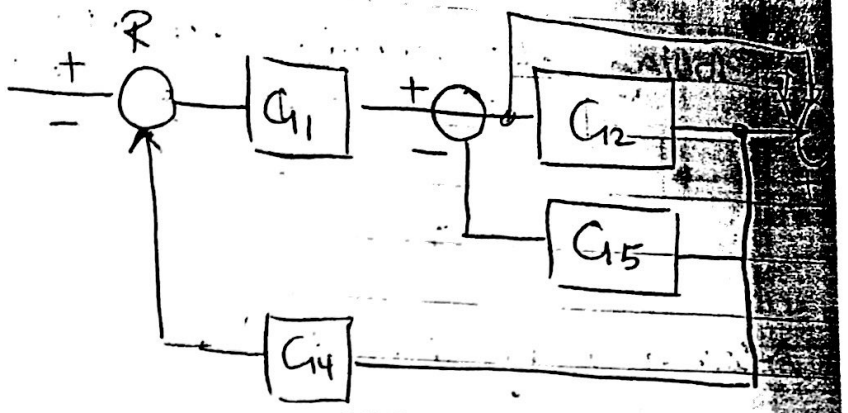


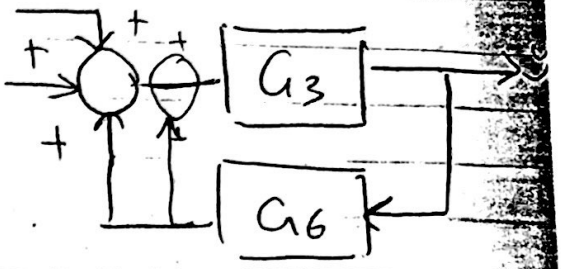
$y/x =$ $1 + G$

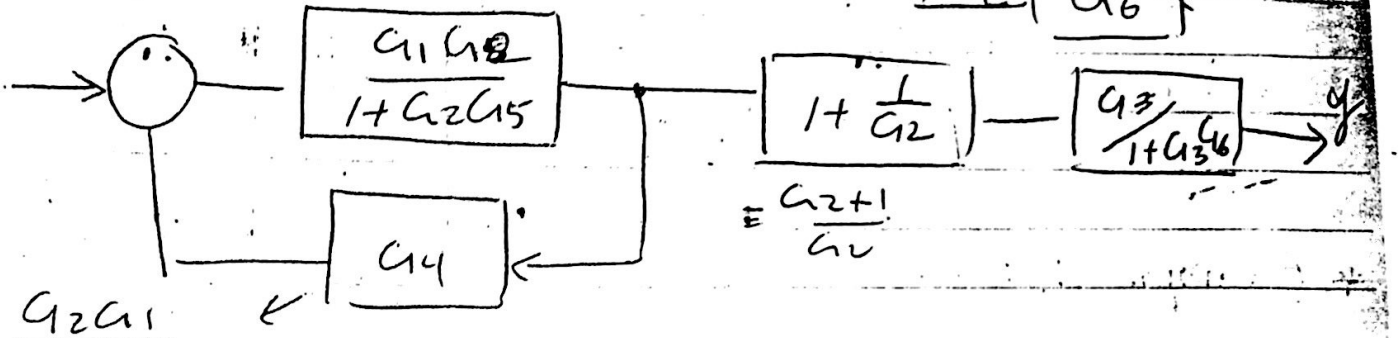
Scanned by CamScanner

* Example (2):-

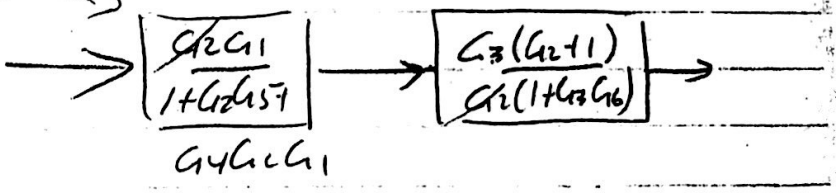
Find y/R







$$\frac{G_2G_1}{1+G_2G_5} \Big/ \frac{1+G_2G_1G_4}{1+G_2G_5}$$



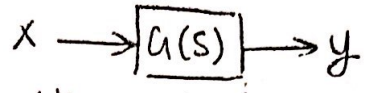
$$\frac{y}{x} = \frac{G_1G_3(G_2+1)}{(1+G_2G_5+G_4G_2G_1)(1+G_3G_6)}$$

5 quiz
5 homework
5 Quiz
15 Test I
10 test II
50. final

Scanned by CamScanner

* Signal flow Graph :-

a signal flow graph consists of nodes directed branches.



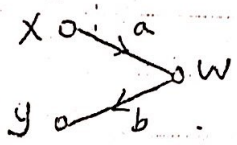
$y = xG(s)$

$\equiv$

$y = Gx$

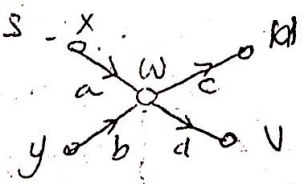
a node performs two functions :-

① - Addition of incoming signals.



$\therefore \quad w = ax + by$

② Transmission of the sum of incoming signals to outgo branches.



$w = ax + by$

$v = wc = (ax + by)c$

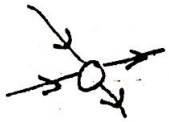
$u = wd = (ax + by)d$

There are 3 types of nodes :-

2016 - 03 - 19

source node

sink node



mixed node

* forward path :-

a branch or continuous sequence of branches from source node to a sink node such that a node appears only once on the branch

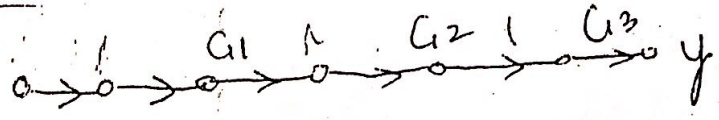


fig (a)

Scanned by CamScanner

P_k = k^{th} forward path Gain :—

The product of branch gains to the k^{th} forword path

for fig (a)

$$P_1 = G_1 G_2 G_3$$

* loop :—

a closed path that starts at a node and ends at the same node

[Fig(b)]

* loop Gain :—

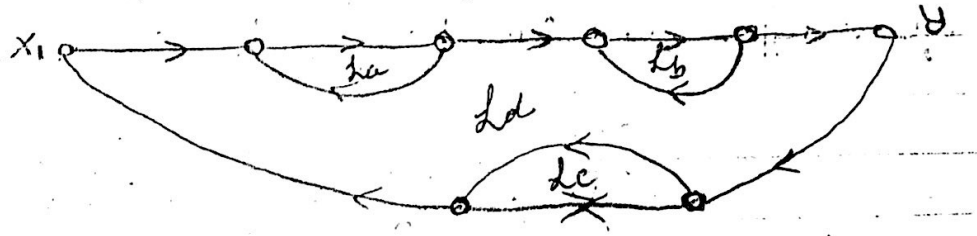
The product of branch gain in a loop.

$$L_1 = G_1 G_2 G_5 \qquad L_2 = G_3 G_4$$

[Fig(b)]:

* Non touching loop :— (a-b) (a-c) (b-c)

two loops are non-touching if they don't have a common node.



* Determinant of the Graph :— (Δ) :-

Δ = 1 − sum of loop gains (La+Lb+Lc) + sum of their product of gains of non-touching loops taken two at a time (LaLb + LbLc + LaLc) − sum of the product of gains of non-touching loops taking 3 at a time (LaLbLc) + ...

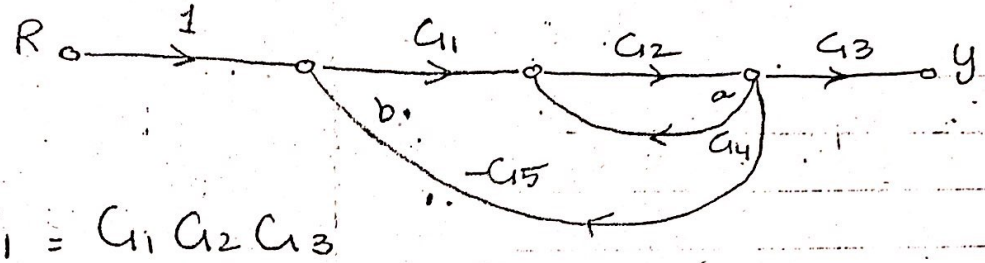
Scanned by CamScanner

Δ_k = Determinant of the remaining graph after removing the loop that touches the kth forward path.

*Mason's Rule :- The transfer function from the input to the output is given by:

$$\frac{y}{R} = G = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 \dots}{\Delta}$$

*Example (1):- Find y/R using Mason's Rule.



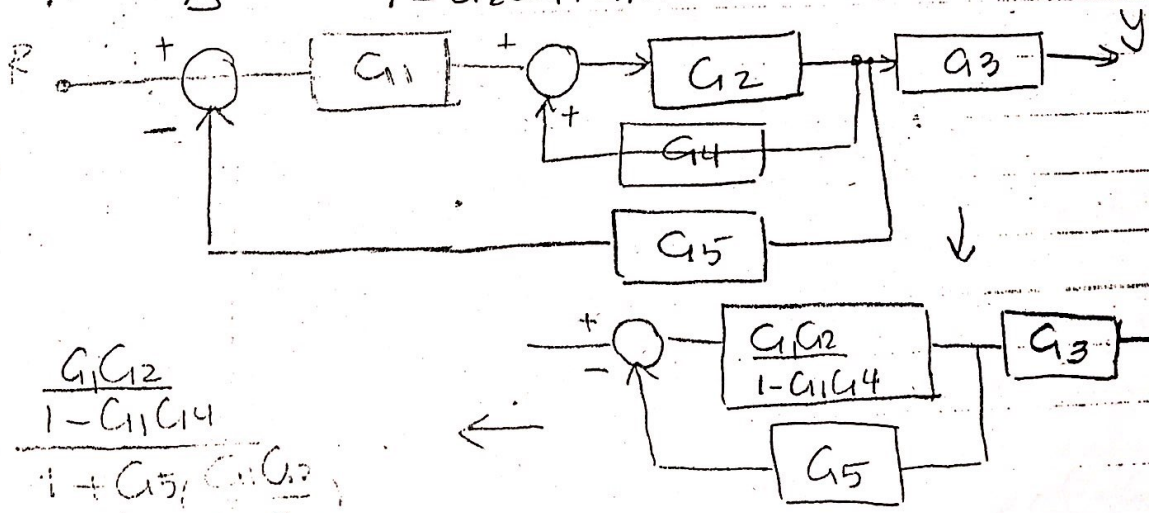
$P_1 = G_1 G_2 G_3$

$L_a = G_2 G_4$ $\quad L_b = -G_1 G_2 G_5$

$\Delta = 1 - (L_a + L_b) + \dots$

$\Delta_1 = 1 - 0$

$$G = \frac{\sum P_k \Delta_k}{\Delta} = \frac{G_1 G_2 G_3 (1)}{1 - G_2 G_4 + G_1 G_2 G_5}$$

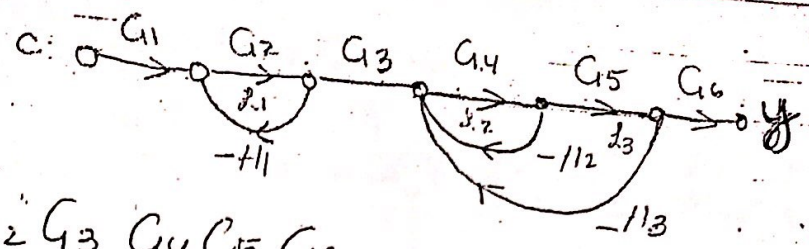


$$G_3 \times \frac{\frac{G_1 G_2}{1 - G_1 G_4}}{1 + G_5 \left(\frac{G_1 G_2}{1 - G_1 G_4}\right)}$$

$$G = \frac{y}{R} = \frac{G_1 G_3 G_2}{1 - G_2 G_4 + G_1 G_2 G_5}$$

Scanned by CamScanner

Example (2) :-



$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$.

$L_1 = -G_2 H_1$.

$L_2 = -G_4 H_2$.

$L_3 = -G_4 G_5 H_3$.

* non touching :-

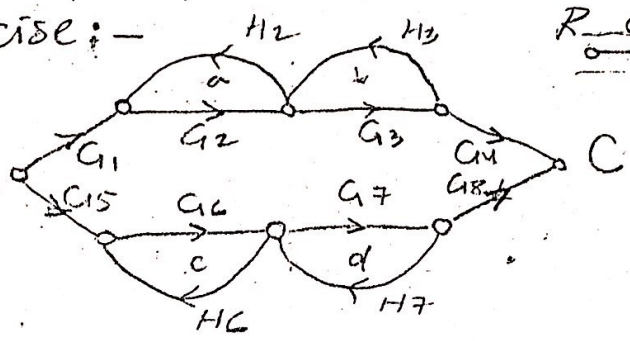
$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3)$

$\Delta_1 = 1 - 0$.

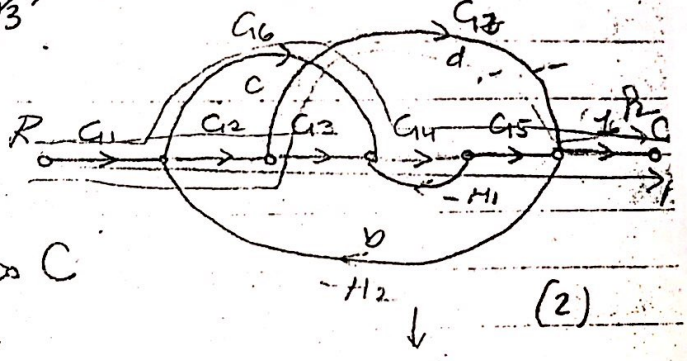
$$\frac{y}{C} = G = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4 G_5 G_6}{1 - (L_1 + L_2 + L_3)(L_1 L_2 + L_1 L_3)}$$

$$= \frac{G_1 G_2 G_3 G_4 G_5 G_6}{1 + G_2 H_1 + G_4 H_2 + G_4 G_5 H_3}$$

* Exercise :-



(1)



(2)

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_1 G_2 G_7$

$P_3 = G_1 G_6 G_4 G_5$

$l_a = -H_1 G_1$

$L_b = -H_2 G_2 G_3 G_4 G_5$

$L_c = -H_2 G_6 G_4 G_5$

$L_d = -H_2 G_2 G_7$

$\Delta = 1 - (L_a + L_b + L_c + L_d) + (L_a L_c$

$\Delta_1 = 1 - 0$

$\Delta_2 = 1 - L_a$

$\Delta_3 = 1 - 0$

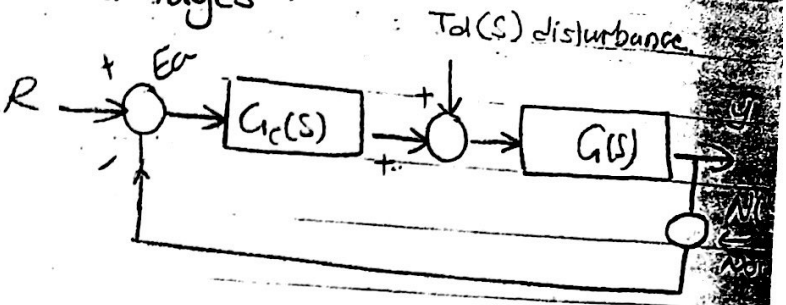
$C =$

Scanned by CamScanner

* feed back System Advantages :-

① Sensetivity.

② Tracking error.



U = Control Signal

E = tracking error. $= R - Y \rightarrow (*)$

$E = R - y.$

$Ea = R - (Y + N(s))$

$$Y = (Td(S) + U)G(S)$$

$$= (Td(S) + Gc(S)Ea)G(s)$$

$$= [Td(S) + Gc(R - (y + N(s)))]G(s)$$

$$Y + (1 + GcG) = GGcR + G\,Td - G\,Gc\,N \quad \ldots -$$

$$Y = \frac{GGc}{1+GGc}R + \frac{G}{1+GGc}Td - \frac{GGc}{1+GGc}N.$$

$$\downarrow$$

$$(*)\ E = R - Y = R - \left(\quad\right)$$

$$= \left(1 - \frac{GGc}{1+GGc}\right)R - \frac{G}{1+GGc}Td + \frac{GGc}{1+GGc}N.$$

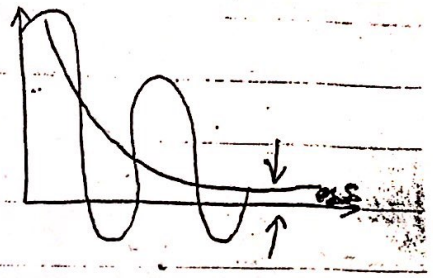
By increasing $Gc(s)$, we reduce the error due to R, and Td

for the Noise we have to Use transfer function to filter the noise.

Scanned by CamScanner

* Steady State Tracking Error :-

$$e_{ss} = e(t=\infty)$$



* Final Value Theorem :-

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

for the pervious figure let: $N=0$, $Td=0$

let $r(t)$ is the unite step $(U(t))$.

$$R(s) = \frac{1}{s^1}$$

$$E(s) = \frac{1}{1+G_c(s)G(s)} R(s)$$

$$e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{s}{1+G_c(s)G(s)} \cdot \frac{1}{s}$$

$$= \frac{1}{1+G_c(0)G(0)}$$

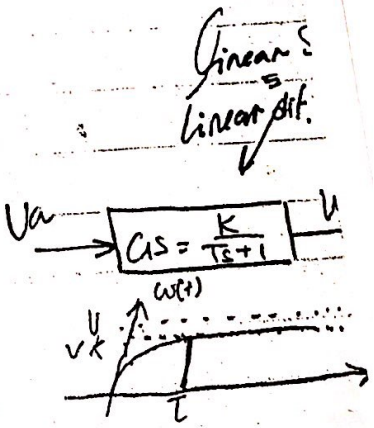
(3) Sharping the transient response:

→ V/s

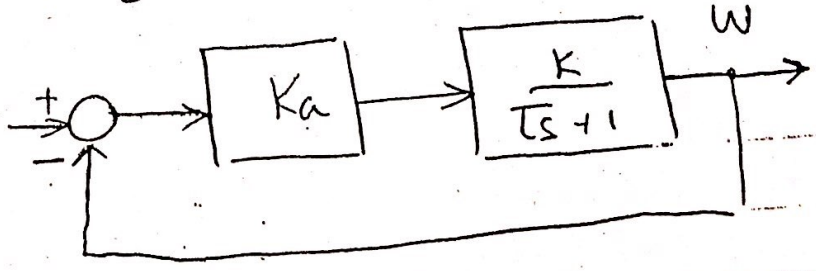
$$V_a(t) = V_u(t)$$

$$W(s) = \frac{KV}{(\tau s+1)s}$$

$$W(t) = KV\left[1-e^{-t/\tau}\right]$$



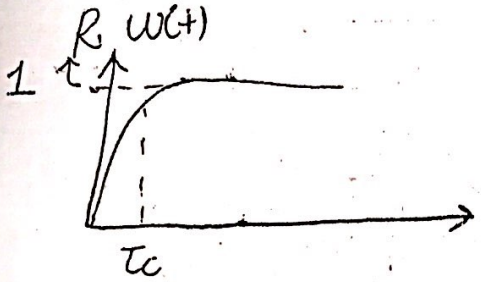
* for : C.I.S :-



Scanned by CamScanner

$$\therefore T = \frac{W}{R} = \frac{\dfrac{K K_a / \tau s + 1}{\quad}}{1 + \left(\dfrac{\quad}{\quad}\right)} = \frac{K K_a}{\tau s + 1 + K K_a}$$

$$= \frac{K K_a / 1 + K K_a}{\dfrac{\tau}{1 + K K_a} s + 1} = \frac{K_c}{\tau_c s + 1}$$



By increasing K_a we reduce τ_c that means faster response which is better.

Scanned by CamScanner